Calculus Warm Up #4-2

Same problem as yesterday...
The radius of a sphere is increasing at a constant rate of 0.04 cm/sec.

At the time when the volume is 36π cm^3 what is the rate of increase of the area of a cross section through the center of the sphere?

HW Questions: p. 598

15) $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{\pi}{2}(2n-1)$

a_1 a_2 a_3 a_4

$\frac{\sin \frac{\pi}{2}}{1}$ $\frac{\sin \frac{3\pi}{2}}{2}$ $\frac{\sin \frac{5\pi}{2}}{3}$ $\frac{\sin \frac{7\pi}{2}}{4}$

$\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}$

$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right)$ $\frac{1}{n+1} \leq \frac{1}{n}$ ✓

$\lim_{n\to\infty} \frac{1}{n} = 0$ ✓

Series converges by the Alt. Series Test

19) $\lim_{n\to\infty} \frac{\sqrt{n}}{n+2} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

$\frac{\sqrt{n+1}}{n+3} \leq \frac{\sqrt{n}}{n+2}$ ✓

10.5 Alternating Series

The Alternating Series Test for Convergence

The Alternating Series Remainder

Absolute Convergence

Conditional Convergence

Estimating the sum of a
Converging Alternating Series:

When $a_{n+1} \leq a_n$, using the partial sum, S_N

to estimate $S = \sum a_n$

The accuracy of the estimate:

$$|S - S_N| = |R_N| \leq a_{N+1}$$

N = Term # of the last term in the series included in the partial sum

R_N = Remainder of the Series (sum of the remaining terms)

Does the series converge? If so,
approximate the sum using the first 6 terms.
How accurate is your result?

$$\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n!}\right)$$

$$\lim_{n \to \infty} \frac{1}{n!} = 0 \checkmark$$

$$= \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} + \cdots$$

$$\boxed{\frac{1}{(n+1)!} \leq \frac{1}{n!} \checkmark}$$

Series converges by the Alternating Series Test.

Approximate the sum using the first 6 terms.
How accurate is your result?

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n!}\right) \qquad \text{Since } \frac{1}{(n+1)!} \leq \frac{1}{n!}$$

$$S \approx S_6 = \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720}$$

$$S_6 = \frac{91}{144} \approx 0.6319$$

How accurate is your result?

The accuracy of the estimate:

$$|S - S_N| = |R_N| \leq a_{N+1}$$

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n!}\right)$$

$$S \approx S_6 = \frac{91}{144} \approx 0.6319$$

Alternating Series Remainder:

$$|R_6| \leq a_{6+1} \qquad a_7 = \frac{1}{7!} = \frac{1}{5040}$$

$$|R_6| \leq \approx 0.0002 \qquad \approx 0.0002$$

$\therefore$ The actual Sum, S, is $\approx$

$$0.6319 - |R_6| \leq S \leq 0.6319 + |R_6|$$

$$0.6317 \leq S \leq 0.6321$$

Sometimes a series has positive and negative terms, but does not alternate. New tools...

Absolute vs Conditional Convergence

Absolute Convergence:

If $\sum |a_n|$ converges, then $\sum a_n$ also converges.

* The converse of this statement is NOT true.

If $\sum a_n$ converges, it does not follow that $\sum |a_n|$ also converges.

Absolute vs Conditional Convergence

Absolute Convergence:

If $\sum |a_n|$ converges, then $\sum a_n$ also converges.

* The converse of this statement is NOT true.

Ex: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ ← Alternating harmonic series

Converges by the Alternating Series Test

But $|a_n| = \frac{1}{n}$

$\sum \frac{1}{n}$ ← Harmonic Series Diverges $(p=1)$

This is called Conditional Convergence

Absolute vs Conditional Convergence

Absolute Convergence:

If $\sum |a_n|$ converges, then $\sum a_n$ also converges.

Conditional Convergence:

$\sum a_n$ is conditionally convergent if,

$\sum a_n$ converges, but $\sum |a_n|$ diverges.

HW: p. 598, # 23 - 45 odd

over 2-days. :)

Group quiz Friday over 10.1 - 10.4